## Explain lower and upper tail test in t and z tests

**ChatGPT said:**

In hypothesis testing, **lower-tail**, **upper-tail**, and **two-tail** tests refer to where in the distribution we look for evidence against the **null hypothesis (H₀)**. This applies to both **t-tests** and **z-tests**, which differ mainly in the type of data and sample size involved:

* **z-test**: Used when population standard deviation is known, and/or sample size is large (n ≥ 30).
* **t-test**: Used when population standard deviation is unknown, and sample size is small (n < 30).

**1. Lower-Tail Test (Left-Tailed Test)**

* **Purpose**: To test if a parameter (like a mean) is significantly **less than** a specified value.
* **Hypotheses**:
  + H₀: μ ≥ μ₀
  + H₁: μ < μ₀
* **Rejection Region**: In the **left tail** of the distribution.
* **Critical value**: Compare test statistic to **−zₐ** or **−tₐ**.

**Example**: You want to check if a battery lasts less than 10 hours.

**2. Upper-Tail Test (Right-Tailed Test)**

* **Purpose**: To test if a parameter is significantly **greater than** a specified value.
* **Hypotheses**:
  + H₀: μ ≤ μ₀
  + H₁: μ > μ₀
* **Rejection Region**: In the **right tail**.
* **Critical value**: Compare test statistic to **zₐ** or **tₐ**.

**Example**: You want to test if a new teaching method increases test scores above the current average.

**3. Two-Tail Test**

* **Purpose**: To test if a parameter is **different** (either higher or lower) from a specified value.
* **Hypotheses**:
  + H₀: μ = μ₀
  + H₁: μ ≠ μ₀
* **Rejection Regions**: Both **tails** of the distribution.
* **Critical value**: Compare test statistic to ±zₐ/2 or ±tₐ/2.

**Example**: You want to test if a new drug changes blood pressure (could go up or down).

**Visualization (Simplified):**

| **Test Type** | **H₁ Direction** | **Rejection Region** |
| --- | --- | --- |
| Lower-Tail | μ < μ₀ | Left tail (test stat < −critical value) |
| Upper-Tail | μ > μ₀ | Right tail (test stat > critical value) |
| Two-Tail | μ ≠ μ₀ | Both tails (test stat > ±critical value) |

**Summary:**

* **Choose test direction** based on your research question (greater than, less than, or not equal).
* Use **z** when σ (population standard deviation) is known and n is large; use **t** when σ is unknown and/or n is small.
* The **p-value** interpretation also depends on the tail: you compare it to α (e.g., 0.05) only in the rejection tail(s).

Let me know if you want an example calculation for each test type.

**You said:**

## lec 28.ppt

Presentation

explain all the topics in the slides in ver details and simple words with examples by dividing topics into good steps

**ChatGPT said:**

Updated saved memory

Here's a **detailed and simple explanation** of all the topics covered in your slides on **Chapter 10: Two-Sample Tests**, broken down step-by-step with examples and interpretations.

**🧪 1. What Are Two-Sample Tests?**

**Goal**: Compare **two population means** to see if there is a significant difference between them.

**Two types of comparisons:**

1. **Independent Samples**: Two different groups (e.g., NYSE vs NASDAQ stocks).
2. **Related Samples (Paired Samples)**: Same group measured twice (e.g., before vs after training).

**🧪 2. Hypothesis Testing for Two Population Means**

You want to test whether **μ₁ (mean of group 1)** is different from **μ₂ (mean of group 2)**.

**➤ Lower-tail test:**

* H₀: μ₁ − μ₂ ≥ 0
* H₁: μ₁ − μ₂ < 0  
  → Used when testing **if μ₁ is less than μ₂**

**➤ Upper-tail test:**

* H₀: μ₁ − μ₂ ≤ 0
* H₁: μ₁ − μ₂ > 0  
  → Used when testing **if μ₁ is greater than μ₂**

**➤ Two-tail test:**

* H₀: μ₁ = μ₂
* H₁: μ₁ ≠ μ₂  
  → Used when testing **if there is any difference** (no direction)

**📊 3. Independent Samples: Formula for Test Statistic**

**If population variances are:**

* **Known** → Use **Z-test**
* **Unknown but equal** → Use **pooled t-test**
* **Unknown and not equal** → Use **Welch's t-test**

**📚 4. Example: Pooled-Variance t-test (Equal Variances)**

**Problem:**

Is there a difference in dividend yield between NYSE and NASDAQ?

|  | **NYSE** | **NASDAQ** |
| --- | --- | --- |
| Sample size | 21 | 25 |
| Mean yield | 3.27 | 2.53 |
| Std. Dev | 1.30 | 1.16 |

**Step-by-step:**

1. **Hypotheses**:
   * H₀: μ₁ − μ₂ = 0 (no difference)
   * H₁: μ₁ − μ₂ ≠ 0 (some difference)
2. **Significance level (α)** = 0.05  
   Degrees of freedom = n₁ + n₂ − 2 = 44  
   Critical t-values = ±2.0154
3. **Calculate test statistic**:  
   t=3.27−2.53(1.302/21)+(1.162/25)≈2.04*t*=(1.302/21)+(1.162/25)​3.27−2.53​≈2.04
4. **Compare with critical values**:  
   Since 2.04 > 2.0154 → Reject H₀
5. **Conclusion**:  
   There **is a significant difference** in dividend yields.

**🔁 5. Related Samples (Paired t-test)**

**When to use:**

* Same people measured before and after (e.g., training effect)

**Key Formula:**

Let **D = X₁ − X₂** (difference for each pair)  
Then:

t=Dˉ−μDsD/n*t*=*sD*​/*n*​*D*ˉ−*μD*​​

Where:

* Dˉ*D*ˉ: average difference
* sD*sD*​: standard deviation of the differences
* n*n*: number of pairs

**📚 6. Example: Paired t-test (Before vs After)**

**Problem**: Does training reduce complaints?

| **Salesperson** | **Before** | **After** | **Difference** |
| --- | --- | --- | --- |
| C.B. | 6 | 4 | -2 |
| T.F. | 20 | 6 | -14 |
| M.H. | 3 | 2 | -1 |
| R.K. | 0 | 0 | 0 |
| M.O. | 4 | 0 | -4 |

* Dˉ=−4.2*D*ˉ=−4.2, total difference = -21
* n=5*n*=5, degrees of freedom = 4
* **Critical t (α = 0.01)** = ±4.604
* **Calculated t** = -1.66

Since -1.66 is not in the rejection region → **Do not reject H₀**

**Conclusion**: Not enough evidence that training reduced complaints.

**💻 7. Running These Tests in Excel**

Use **Data Analysis Toolpak**:

* **Z-test**: “Z-Test: Two Sample for Means”
* **Pooled t-test**: “t-Test: Two Sample Assuming Equal Variances”
* **Welch’s t-test**: “t-Test: Two Sample Assuming Unequal Variances”
* **Paired t-test**: “t-Test: Paired Two Sample for Means”
* **Variance test (F-test)**: “F-Test: Two-Sample for Variances”

**✅ Summary Table:**

| **Test Type** | **Use When** | **Example** |
| --- | --- | --- |
| Z-Test | σ known or large n | Compare two product quality scores |
| Pooled t-test | σ unknown, assumed equal | Compare average yield between NYSE & NASDAQ |
| Welch’s t-test | σ unknown, not equal | Compare GPA from two universities |
| Paired t-test | Same subject measured twice | Before-and-after training performance |

Let me know if you'd like practice problems, Excel screenshots, or a cheat sheet!

**You said:**

4. Example: Pooled-Variance t-test (Equal Variances) Problem: Is there a difference in dividend yield between NYSE and NASDAQ? NYSE NASDAQ Sample size 21 25 Mean yield 3.27 2.53 Std. Dev 1.30 1.16 Step-by-step: Hypotheses: H₀: μ₁ − μ₂ = 0 (no difference) H₁: μ₁ − μ₂ ≠ 0 (some difference) Significance level (α) = 0.05 Degrees of freedom = n₁ + n₂ − 2 = 44 Critical t-values = ±2.0154 Calculate test statistic: 𝑡 = 3.27 − 2.53 ( 1.30 2 / 21 ) + ( 1.16 2 / 25 ) ≈ 2.04 t= (1.30 2 /21)+(1.16 2 /25) ​ 3.27−2.53 ​ ≈2.04 Compare with critical values: Since 2.04 > 2.0154 → Reject H₀ Conclusion: There is a significant difference in dividend yields. explain this in details

**ChatGPT said:**

Sure! Let's go through this example **step by step**, in **very simple words**, and explain **what’s happening and why** at each step.

**🎯 Goal of the Test**

We want to **compare the average (mean) dividend yields** of two types of stocks:

* **NYSE stocks** (New York Stock Exchange)
* **NASDAQ stocks**

We’re checking if the **difference in averages** is **real (statistically significant)** or just due to random chance.

**📊 Given Data**

|  | **NYSE** | **NASDAQ** |
| --- | --- | --- |
| Sample size (n) | 21 | 25 |
| Mean yield (X̄) | 3.27 | 2.53 |
| Std. deviation (s) | 1.30 | 1.16 |

**🧪 Step 1: State Hypotheses**

We write two statements:

* **Null Hypothesis (H₀):**  
  No difference in average dividend yields.  
  → H₀: μ₁ − μ₂ = 0
* **Alternative Hypothesis (H₁):**  
  There **is** a difference (either higher or lower).  
  → H₁: μ₁ − μ₂ ≠ 0  
  This is a **two-tailed test**, because we’re checking for **any difference**, not just higher or lower.

**🎯 Step 2: Choose Significance Level**

Let’s set:

* **α = 0.05** (This means we allow a 5% chance of being wrong if we reject H₀)
* **Degrees of Freedom (df)** = 21 + 25 − 2 = 44
* **Critical t-values** for df = 44 at α = 0.05 (two-tailed): ±2.0154  
  → If our test result (t) is **more extreme** than ±2.0154, we’ll **reject H₀**.

**🧮 Step 3: Calculate the Test Statistic (t)**

We use the formula for **pooled-variance t-test** (equal variances assumed):

t=Xˉ1−Xˉ2s12n1+s22n2*t*=*n*1​*s*12​​+*n*2​*s*22​​​*X*ˉ1​−*X*ˉ2​​

Now plug in the numbers:

* Xˉ1=3.27*X*ˉ1​=3.27, Xˉ2=2.53*X*ˉ2​=2.53
* s1=1.30*s*1​=1.30, s2=1.16*s*2​=1.16
* n1=21*n*1​=21, n2=25*n*2​=25

**Step-by-step math**:

1. Square the standard deviations:
   * 1.302=1.691.302=1.69
   * 1.162=1.34561.162=1.3456
2. Divide by sample sizes:
   * 1.69/21≈0.08051.69/21≈0.0805
   * 1.3456/25≈0.05381.3456/25≈0.0538
3. Add them:
   * 0.0805+0.0538=0.13430.0805+0.0538=0.1343
4. Square root:
   * 0.1343≈0.36650.1343​≈0.3665
5. Subtract means and divide:
   * (3.27−2.53)=0.74(3.27−2.53)=0.74
   * 0.74/0.3665≈2.020.74/0.3665≈2.02

✔️ Final **t-value ≈ 2.02** (slides round it to **2.04**, so close enough).

**🔍 Step 4: Compare With Critical Value**

* Our calculated **t = 2.04**
* The **critical value = ±2.0154**

Since:

∣2.04∣>2.0154∣2.04∣>2.0154

→ The result **falls in the rejection region**.

**✅ Step 5: Conclusion**

* **We reject H₀**.
* There is enough statistical evidence to say the **average dividend yields for NYSE and NASDAQ are different**.

**🚨 Important:**

This doesn’t tell us which one is better — only that **they are not the same**. However, since NYSE has a **higher mean (3.27 vs. 2.53)**, it **appears** to offer higher yields **on average**.

**📌 Summary Table**

| **Step** | **Action** | **Result** |
| --- | --- | --- |
| 1 | Set H₀ and H₁ | H₀: μ₁ = μ₂, H₁: μ₁ ≠ μ₂ |
| 2 | Set α and find critical values | α = 0.05 → t = ±2.0154 |
| 3 | Calculate t-statistic | t ≈ 2.04 |
| 4 | Compare t to critical value | 2.04 > 2.0154 → Reject H₀ |
| 5 | Final conclusion | Difference **is** significant |

Let me know if you want help doing this in **Excel**, or want a **one-tail version** of the test explained too.

**You said:**

can you provides examples onf all types of t tests

**ChatGPT said:**

Absolutely! Here's a **clear explanation and example for each type of t-test**, broken down into four types commonly covered in business statistics:

**📌 Overview of Types of t-Tests:**

| **Test Type** | **When to Use** |
| --- | --- |
| 1. One-Sample t-Test | Compare sample mean to a known population mean |
| 2. Independent Samples t-Test (Equal/Unequal Variance) | Compare means of two **different** groups |
| 3. Paired Samples t-Test | Compare means from **same group** before and after |

**🔹 1. One-Sample t-Test**

**🧪 When: You have a sample and want to test if its mean is different from a known population mean.**

**📘 Example:**

A call center claims that the average response time is **5 minutes**. A sample of **15 calls** shows an average time of **5.4 minutes**, with a sample standard deviation of **0.8 minutes**.

* H₀: μ = 5
* H₁: μ ≠ 5
* n = 15, X̄ = 5.4, s = 0.8

t=5.4−50.8/15=0.40.2066≈1.94*t*=0.8/15​5.4−5​=0.20660.4​≈1.94

Compare t to critical value at df = 14, α = 0.05 → ±2.145. Since 1.94 < 2.145 → **Do not reject H₀**.

✅ **Conclusion**: Not enough evidence to say the average is different from 5.

**🔹 2. Independent Samples t-Test**

This test has **two versions** depending on variance:

**2a. Equal Variances (Pooled t-Test)**

**🧪 When: You have two independent groups and assume equal population variances.**

**📘 Example:**

Are average test scores different for students from two schools?

| **Group** | **n** | **X̄** | **s** |
| --- | --- | --- | --- |
| School A | 20 | 78 | 5 |
| School B | 22 | 82 | 4.8 |

* H₀: μ₁ = μ₂
* H₁: μ₁ ≠ μ₂

t=78−825220+4.8222≈−41.52≈−2.63*t*=2052​+224.82​​78−82​≈1.52−4​≈−2.63

At df = 20 + 22 − 2 = 40, critical t = ±2.021 → **Reject H₀**

✅ **Conclusion**: There is a significant difference in test scores.

**2b. Unequal Variances (Welch’s t-Test)**

**🧪 When: Two independent groups but variances are not equal.**

Same example as above, but suppose the variances are clearly different. Use:

t=Xˉ1−Xˉ2s12n1+s22n2*t*=*n*1​*s*12​​+*n*2​*s*22​​​*X*ˉ1​−*X*ˉ2​​

The formula is the same, but the **degrees of freedom are calculated differently** using a more complex formula (Welch-Satterthwaite).

If t ≈ -2.63 and df ≈ 38 → Still **reject H₀** at α = 0.05.

**🔹 3. Paired Samples t-Test (Dependent Samples)**

**🧪 When: The same subjects are measured before and after an event.**

**📘 Example:**

A company trains 5 employees and tracks number of complaints **before and after** training.

| **Employee** | **Before** | **After** | **D (Diff)** |
| --- | --- | --- | --- |
| A | 5 | 3 | 2 |
| B | 7 | 5 | 2 |
| C | 6 | 3 | 3 |
| D | 4 | 2 | 2 |
| E | 5 | 5 | 0 |

* Mean Difference D̄ = 1.8, SD of differences ≈ 1.095
* H₀: μ\_D = 0, H₁: μ\_D ≠ 0

t=1.8−01.095/5≈1.80.49≈3.67*t*=1.095/5​1.8−0​≈0.491.8​≈3.67

df = 4, critical t ≈ ±2.776 → **Reject H₀**

✅ **Conclusion**: Training significantly reduced complaints.

**✅ Summary Table**

| **Test Type** | **When to Use** | **Sample Size** | **Example** |
| --- | --- | --- | --- |
| One-Sample t-Test | Compare sample mean to population mean | One group | Check if avg time ≠ 5 mins |
| Two-Sample t-Test (Equal Var) | Compare two independent group means (equal variances) | Two groups | NYSE vs NASDAQ yields |
| Two-Sample t-Test (Unequal Var) | Same as above, but variances not equal | Two groups | GPA from different universities |
| Paired Samples t-Test | Same group tested twice | Same group twice | Complaints before vs after training |

Let me know if you want any of these worked out **in Excel** or with visual graphs!

**You said:**

can you tell how to solve two sample t test and t tets what is there formula how to calculate there df step by step

**ChatGPT said:**

Absolutely! Let's go over how to **solve a two-sample t-test** in a **step-by-step way**, including:

1. **When to use it**
2. **Which formula to use**
3. **How to calculate degrees of freedom (df)**
4. **Step-by-step example**

**✅ TWO-SAMPLE t-TEST: What Is It?**

Used when you want to compare the **means of two independent groups** (e.g., Group A vs Group B).

**📌 You use this test when:**

* The groups are **independent**
* You're comparing their **means (averages)**
* Population variances **may or may not be equal**

**📚 There are TWO types of two-sample t-tests:**

| **Type** | **Assumption** | **Formula** |
| --- | --- | --- |
| 1. **Pooled t-test** | Equal variances (σ₁² = σ₂²) | Use **pooled variance** |
| 2. **Welch’s t-test** | Unequal variances (σ₁² ≠ σ₂²) | Use **unpooled formula** |

**🔹 FORMULAS**

**1. Pooled t-test (equal variances)**

**🧪 Formula:**

t=Xˉ1−Xˉ2Sp2(1n1+1n2)*t*=*Sp*2​(*n*1​1​+*n*2​1​)​*X*ˉ1​−*X*ˉ2​​

Where:

* Sp2*Sp*2​ = pooled variance

Sp2=(n1−1)S12+(n2−1)S22n1+n2−2*Sp*2​=*n*1​+*n*2​−2(*n*1​−1)*S*12​+(*n*2​−1)*S*22​​

**🧮 Degrees of freedom (df):**

df=n1+n2−2*df*=*n*1​+*n*2​−2

**2. Welch’s t-test (unequal variances)**

**🧪 Formula:**

t=Xˉ1−Xˉ2S12n1+S22n2*t*=*n*1​*S*12​​+*n*2​*S*22​​​*X*ˉ1​−*X*ˉ2​​

**🧮 Degrees of freedom:**

df=(S12n1+S22n2)2(S12n1)2n1−1+(S22n2)2n2−1*df*=*n*1​−1(*n*1​*S*12​​)2​+*n*2​−1(*n*2​*S*22​​)2​(*n*1​*S*12​​+*n*2​*S*22​​)2​

This df is usually **non-integer**, and software (or calculator) is recommended.

**✅ STEP-BY-STEP SOLUTION (Pooled t-Test Example)**

**Example**:  
A researcher wants to compare two groups:

| **Group** | **n** | **Mean (X̄)** | **Std Dev (s)** |
| --- | --- | --- | --- |
| Group A | 10 | 85 | 5 |
| Group B | 12 | 80 | 6 |

We assume equal variances.

**🔢 Step 1: State the Hypotheses**

* H₀: μ₁ = μ₂ → No difference
* H₁: μ₁ ≠ μ₂ → There **is** a difference

**🔢 Step 2: Set significance level**

* α = 0.05 (default)
* df = 10 + 12 − 2 = 20
* Critical t (from table) = ±2.086 (two-tailed test)

**🔢 Step 3: Calculate pooled variance (Sp²)**

Sp2=(10−1)(52)+(12−1)(62)10+12−2=9(25)+11(36)20=225+39620=62120=31.05*Sp*2​=10+12−2(10−1)(52)+(12−1)(62)​=209(25)+11(36)​=20225+396​=20621​=31.05

**🔢 Step 4: Compute the test statistic**

t=85−8031.05(110+112)*t*=31.05(101​+121​)​85−80​

First calculate:

110+112=0.1+0.0833=0.1833101​+121​=0.1+0.0833=0.1833t=531.05×0.1833=55.69≈52.385≈2.10*t*=31.05×0.1833​5​=5.69​5​≈2.3855​≈2.10

**🔢 Step 5: Compare t to critical value**

* t = 2.10
* Critical value = ±2.086
* Since **2.10 > 2.086**, we **reject H₀**

**✅ Conclusion:**

There **is a significant difference** between the two group means at the 0.05 level.

**💡 TIP: How to Decide Which Test?**

| **Situation** | **Test Type** |
| --- | --- |
| Same people, before/after | Paired t-test |
| Two different groups, same σ | Pooled t-test |
| Two different groups, diff σ | Welch's t-test |
| One group vs. known value | One-sample t-test |

Would you like a **step-by-step Excel guide** or calculator-based solution next?

**You said:**

t= S p 2 ​ ( n 1 ​ 1 ​ + n 2 ​ 1 ​ ) ​ there is a diiff bt this one and my slides oone

**ChatGPT said:**

Great observation! Yes — there **appears to be a difference in the formula format** between how the **two-sample t-test** formula is written in your **slides** and the version I gave earlier.

Let’s carefully break this down to explain **why they look different but mean the same thing**.

**🔄 The Two Formats You're Seeing**

**✅ Your Slide’s Format (Pooled t-test):**

The slide shows the t-statistic as:

t=Xˉ1−Xˉ2Sp2(1n1+1n2)*t*=*Sp*2​(*n*1​1​+*n*2​1​)​*X*ˉ1​−*X*ˉ2​​

This is **exactly the correct and standard pooled t-test formula**.

**✅ Alternate Expanded Format You Mentioned:**

It seems you’re referring to something that looks like:

t=Xˉ1−Xˉ2(S12n1+S22n2)*t*=(*n*1​*S*12​​+*n*2​*S*22​​)​*X*ˉ1​−*X*ˉ2​​

This version is used when the **population variances are assumed to be unequal**. That’s the **Welch's t-test** version.

**✅ So What’s the Actual Difference?**

| **Formula** | **Used For** | **Variances** | **Key Feature** |
| --- | --- | --- | --- |
| t=Xˉ1−Xˉ2Sp2(1n1+1n2)*t*=*Sp*2​(*n*1​1​+*n*2​1​)​*X*ˉ1​−*X*ˉ2​​ | **Pooled t-test** | Equal | Uses **pooled variance** Sp² |
| t=Xˉ1−Xˉ2(S12n1+S22n2)*t*=(*n*1​*S*12​​+*n*2​*S*22​​)​*X*ˉ1​−*X*ˉ2​​ | **Welch’s t-test** | Unequal | Uses **individual sample variances** |

**📌 Important:**

The structure of the denominator changes depending on whether you assume the two populations have the **same variance** or **different variances**.

If your slides are using the **first version**, they are assuming:

* The two populations are **independent**
* The variances are **equal**
* Hence, they use the **pooled-variance t-test**

**🔁 Recap:**

**If variances are equal (your slide):**

t=Xˉ1−Xˉ2Sp2(1n1+1n2)*t*=*Sp*2​(*n*1​1​+*n*2​1​)​*X*ˉ1​−*X*ˉ2​​

Where:

* Sp2=(n1−1)S12+(n2−1)S22n1+n2−2*Sp*2​=*n*1​+*n*2​−2(*n*1​−1)*S*12​+(*n*2​−1)*S*22​​
* Degrees of freedom df=n1+n2−2*df*=*n*1​+*n*2​−2

**If variances are unequal (alternative method):**

t=Xˉ1−Xˉ2(S12n1+S22n2)*t*=(*n*1​*S*12​​+*n*2​*S*22​​)​*X*ˉ1​−*X*ˉ2​​

* Used in **Welch’s t-test**
* Degrees of freedom calculated using a **longer formula**

**🧠 So What Should You Use?**

Use the **version from your slides** (pooled t-test) **only if** you're told or can assume **equal population variances**.

If you're not sure whether variances are equal, or your data shows a big difference in sample variances, you should **use the unequal variance (Welch's) formula**.

Would you like a visual flowchart to help you choose the right t-test every time?

**You said:**

no my slide contain u1-u1 meu

**ChatGPT said:**

Ah, thank you for the clarification — you're pointing out that your slide uses the notation **μ₁ − μ₂** (population means) in the hypothesis and formulas. Let's explain what that means and how it fits into the full picture.

**✅ Clarifying the Notation: μ₁ − μ₂ vs X̄₁ − X̄₂**

In your slide, you're seeing something like:

t=(Xˉ1−Xˉ2)−(μ1−μ2)Standard Error*t*=Standard Error(*X*ˉ1​−*X*ˉ2​)−(*μ*1​−*μ*2​)​

And this may seem different from:

t=Xˉ1−Xˉ2Standard Error*t*=Standard Error*X*ˉ1​−*X*ˉ2​​

But here's the key:

**🎯 In hypothesis testing, we often assume:**

H0:μ1−μ2=0*H*0​:*μ*1​−*μ*2​=0

So under **H₀**, the formula simplifies to:

t=(Xˉ1−Xˉ2)−0Standard Error=Xˉ1−Xˉ2Standard Error*t*=Standard Error(*X*ˉ1​−*X*ˉ2​)−0​=Standard Error*X*ˉ1​−*X*ˉ2​​

**✅ So both formulas are the same — the only difference is:**

* Your slide **includes the full general form**:  
  t=(Xˉ1−Xˉ2)−(μ1−μ2)SE*t*=SE(*X*ˉ1​−*X*ˉ2​)−(*μ*1​−*μ*2​)​
* In practice, we assume μ1−μ2=0*μ*1​−*μ*2​=0 under the null hypothesis, so it becomes:  
  t=Xˉ1−Xˉ2SE*t*=SE*X*ˉ1​−*X*ˉ2​​

**📌 Example with Both Forms**

Let’s say:

* X̄₁ = 3.27, X̄₂ = 2.53
* H₀: μ₁ − μ₂ = 0
* SE = 0.36

**Using full version:**

t=(3.27−2.53)−00.36=0.740.36≈2.06*t*=0.36(3.27−2.53)−0​=0.360.74​≈2.06

**Using simplified version:**

t=3.27−2.530.36=2.06*t*=0.363.27−2.53​=2.06

Same result.

**✅ Summary**

| **Notation** | **Meaning** | **Used When** |
| --- | --- | --- |
| (Xˉ1−Xˉ2)−(μ1−μ2)(*X*ˉ1​−*X*ˉ2​)−(*μ*1​−*μ*2​) | General form of test statistic | Theoretical formula |
| Xˉ1−Xˉ2*X*ˉ1​−*X*ˉ2​ only | Used when μ1−μ2=0*μ*1​−*μ*2​=0 under H₀ | Practical testing |

So your slide isn't different — it’s just **showing the full formula with μ₁ − μ₂**, which is **usually 0** in most hypothesis testing problems.

Would you like a practice question where we apply this version step-by-step?

**You said:**

means how can we write the two hypothe

**ChatGPT said:**

Great question!

When you're writing hypotheses for a **two-sample t-test**, you’re testing whether the **population means** of two groups are different (or one is greater than/less than the other). This is expressed using **μ₁ and μ₂**, which represent the **true (unknown) means** of the two populations.

**✅ General Form of Hypotheses**

| **Type of Test** | **Null Hypothesis (H₀)** | **Alternative Hypothesis (H₁)** | **Meaning** |
| --- | --- | --- | --- |
| Two-tail test | H₀: μ₁ = μ₂ or μ₁ − μ₂ = 0 | H₁: μ₁ ≠ μ₂ or μ₁ − μ₂ ≠ 0 | Means are **not equal** |
| Lower-tail | H₀: μ₁ ≥ μ₂ or μ₁ − μ₂ ≥ 0 | H₁: μ₁ < μ₂ or μ₁ − μ₂ < 0 | μ₁ is **less than** μ₂ |
| Upper-tail | H₀: μ₁ ≤ μ₂ or μ₁ − μ₂ ≤ 0 | H₁: μ₁ > μ₂ or μ₁ − μ₂ > 0 | μ₁ is **greater than** μ₂ |

**📝 How to Write It (Examples)**

**🔹 Example 1: Two-tailed**

You're testing if the mean dividend yield for NYSE stocks is **different** from NASDAQ stocks.

H₀: μNYSE=μNASDAQorμ1−μ2=0H₁: μNYSE≠μNASDAQorμ1−μ2≠0H₀: H₁: ​*μNYSE*​=*μNASDAQ*​or*μ*1​−*μ*2​=0*μNYSE*​=*μNASDAQ*​or*μ*1​−*μ*2​=0​

**🔹 Example 2: Lower-tailed**

You're testing if a new process reduces the average time compared to the old one.

H₀: μnew≥μoldorμ1−μ2≥0H₁: μnew<μoldorμ1−μ2<0H₀: H₁: ​*μnew*​≥*μold*​or*μ*1​−*μ*2​≥0*μnew*​<*μold*​or*μ*1​−*μ*2​<0​

**🔹 Example 3: Upper-tailed**

You want to test if a new teaching method increases average test scores.

H₀: μnew≤μoldorμ1−μ2≤0H₁: μnew>μoldorμ1−μ2>0H₀: H₁: ​*μnew*​≤*μold*​or*μ*1​−*μ*2​≤0*μnew*​>*μold*​or*μ*1​−*μ*2​>0​

**✏️ Tip:**

You can always write hypotheses in two equivalent ways:

* **μ₁ = μ₂** and **μ₁ ≠ μ₂**
* **μ₁ − μ₂ = 0** and **μ₁ − μ₂ ≠ 0**

They mean the **same thing** — choose whichever fits best with your test formula or your preference.

Would you like a printable summary table for all hypothesis formats?

**You said:**

what s square meanss

**ChatGPT said:**

Great question! Let’s explain it very simply:

**✅ What Does "Squared" Mean?**

The **square** of a number means:

**Multiply the number by itself**.

**✳️ Notation:**

We write the square of a number **with a small 2 as an exponent**:

a2=a×a*a*2=*a*×*a*

**🧮 Examples:**

| **Number** | **Squared** | **Calculation** |
| --- | --- | --- |
| 2 | 2² = 4 | 2 × 2 |
| 5 | 5² = 25 | 5 × 5 |
| 1.3 | 1.3² ≈ 1.69 | 1.3 × 1.3 |
| 6 | 6² = 36 | 6 × 6 |

**🧪 In Statistics:**

You’ll often see **s²**, like:

* **s²**: This means the **sample variance** (how spread out the data is).
* **(X̄₁ − X̄₂)²**: This is the **square of the difference between two means**.

**🔍 Example in a Formula:**

Let’s say:

* Sample standard deviation (s) = 4
* Then variance (s²) = 4² = 16

If a formula uses:

s2/n*s*2/*n*

It means:

* Take the **square** of the standard deviation, then
* **Divide by the sample size**.

**🔁 In words:**

"Square" just means **multiply it by itself**.

Would you like a visual explanation with graphs or a real-life example (like area of a square)?

**You said:**

s square in tetst above

**ChatGPT said:**

Ah, now I see exactly what you're asking — you're referring to the **s²** terms in the **t-test formulas** we discussed earlier.

Let me explain clearly what **s²** means **in the t-test context**.

**✅ What is s² in a t-test?**

* **s²** is the **sample variance**
* It is the **square of the sample standard deviation (s)**

s2=variance=∑(xi−xˉ)2n−1*s*2=variance=*n*−1∑(*xi*​−*x*ˉ)2​

**📌 In a Two-Sample t-Test:**

You use **s₁²** and **s₂²** as the **variances** from each of the two samples.

**So in formulas like:**

t=Xˉ1−Xˉ2s12n1+s22n2*t*=*n*1​*s*12​​+*n*2​*s*22​​​*X*ˉ1​−*X*ˉ2​​

* s12*s*12​ = variance of sample 1
* s22*s*22​ = variance of sample 2
* n1*n*1​, n2*n*2​ = sample sizes

**🔍 Why do we square the standard deviation?**

In the t-test, we are comparing two sample **means**, and we need to estimate how much **random error or variability** there is. This variability is expressed using **variance (s²)**, which makes the math work when combining different samples.

**🧮 Small Example:**

Let’s say:

* Sample 1: s1=5*s*1​=5 → s12=25*s*12​=25
* Sample 2: s2=4*s*2​=4 → s22=16*s*22​=16

Sample sizes:

* n1=10*n*1​=10, n2=12*n*2​=12

Then:

Standard Error=2510+1612=2.5+1.333=3.833≈1.96Standard Error=1025​+1216​​=2.5+1.333​=3.833​≈1.96

So:

* **You square the standard deviations**
* Then **plug the variances into the t-test formula**

**✅ Summary:**

| **Symbol** | **Meaning** | **Role in t-test** |
| --- | --- | --- |
| s | Sample standard deviation | Measure of spread |
| s² | Sample variance (squared s) | Used to calculate **standard error** in t-tests |

Let me know if you'd like me to show how this works in Excel or plug in numbers from your slides!

**You said:**

5. Related Samples (Paired t-test) When to use: Same people measured before and after (e.g., training effect) Key Formula: Let D = X₁ − X₂ (difference for each pair) Then: 𝑡 = 𝐷 ˉ − 𝜇 𝐷 𝑠 𝐷 / 𝑛 t= s D ​ / n ​ D ˉ −μ D ​ ​ Where: 𝐷 ˉ D ˉ : average difference 𝑠 𝐷 s D ​ : standard deviation of the differences 𝑛 n: number of pairs 📚 6. Example: Paired t-test (Before vs After) Problem: Does training reduce complaints? Salesperson Before After Difference C.B. 6 4 -2 T.F. 20 6 -14 M.H. 3 2 -1 R.K. 0 0 0 M.O. 4 0 -4 𝐷 ˉ = − 4.2 D ˉ =−4.2, total difference = -21 𝑛 = 5 n=5, degrees of freedom = 4 Critical t (α = 0.01) = ±4.604 Calculated t = -1.66 Since -1.66 is not in the rejection region → Do not reject H₀ Conclusion: Not enough evidence that training reduced complaints. explain this in details

**ChatGPT said:**

Absolutely! Let’s walk through the **Paired t-test example** in your slides in **very simple, detailed steps** — so you understand exactly **how it works**, **why it's used**, and **what the numbers mean**.

**🧪 What is a Paired t-test?**

A **Paired t-test** is used when:

* You **measure the same people twice** (before and after something), or
* You have **naturally matched pairs** (like twins, or left/right hand performance)

**📘 Purpose:**

To check if there is a **significant average difference** between the two related sets of values.

**📊 Example Problem: Does Training Reduce Complaints?**

A company wants to know if a **customer service training workshop** helped reduce complaints. They measure the **number of complaints** for **5 salespeople**, **before and after** training.

**📝 Step 1: Write the Data Table**

| **Salesperson** | **Before** | **After** | **Difference (D = After - Before)** |
| --- | --- | --- | --- |
| C.B. | 6 | 4 | -2 |
| T.F. | 20 | 6 | -14 |
| M.H. | 3 | 2 | -1 |
| R.K. | 0 | 0 | 0 |
| M.O. | 4 | 0 | -4 |

**🧮 Step 2: Calculate the Differences**

We already have the differences:

D1=−2,D2=−14,D3=−1,D4=0,D5=−4*D*1​=−2,*D*2​=−14,*D*3​=−1,*D*4​=0,*D*5​=−4

**🧮 Step 3: Calculate the Mean Difference**

Dˉ=−2+(−14)+(−1)+0+(−4)5=−215=−4.2*D*ˉ=5−2+(−14)+(−1)+0+(−4)​=5−21​=−4.2

**🧮 Step 4: Calculate the Standard Deviation of Differences**

We use the formula:

sD=∑(Di−Dˉ)2n−1*sD*​=*n*−1∑(*Di*​−*D*ˉ)2​​

Let’s calculate each (Di−Dˉ)2(*Di*​−*D*ˉ)2:

| **D** | **Di−(−4.2)*Di*​−(−4.2)** | **(Di−Dˉ)2(*Di*​−*D*ˉ)2** |
| --- | --- | --- |
| -2 | 2.2 | 4.84 |
| -14 | -9.8 | 96.04 |
| -1 | 3.2 | 10.24 |
| 0 | 4.2 | 17.64 |
| -4 | 0.2 | 0.04 |

∑(Di−Dˉ)2=4.84+96.04+10.24+17.64+0.04=128.8∑(*Di*​−*D*ˉ)2=4.84+96.04+10.24+17.64+0.04=128.8sD=128.85−1=32.2≈5.67*sD*​=5−1128.8​​=32.2​≈5.67

**🧪 Step 5: Use the t-test Formula**

t=Dˉ−μDsD/n*t*=*sD*​/*n*​*D*ˉ−*μD*​​

Assuming **null hypothesis H₀: μ\_D = 0** (no difference), then:

t=−4.2−05.67/5=−4.25.67/2.236=−4.22.535≈−1.66*t*=5.67/5​−4.2−0​=5.67/2.236−4.2​=2.535−4.2​≈−1.66

**✅ Step 6: State Hypotheses**

* **H₀ (null)**: μ\_D = 0 → No change in complaints after training
* **H₁ (alternative)**: μ\_D ≠ 0 → There **is** a change (reduction or increase)

Since this is **two-tailed**, we use **±t** critical values.

**🔍 Step 7: Use Degrees of Freedom and t-table**

* n = 5, so degrees of freedom = 5 − 1 = 4
* α = 0.01 (very strict test)
* Critical t = ±4.604

**❌ Step 8: Make the Decision**

* Calculated t = **−1.66**
* Critical t = **±4.604**

Since −1.66 is **not** in the rejection region (not more extreme than −4.604), we **do not reject H₀**.

**📌 Final Conclusion:**

❌ The training **did not significantly reduce** the number of complaints (at the 0.01 level).  
➡️ The average difference (−4.2) could be due to **random chance**.

**🧠 Summary Table**

| **Step** | **Result** |
| --- | --- |
| Mean Difference | −4.2 |
| Std. Dev (s\_D) | ≈ 5.67 |
| Sample size (n) | 5 |
| t-statistic | −1.66 |
| Critical t | ±4.604 |
| Decision | **Do not reject** H₀ |
| Conclusion | No significant change detected |

Would you like me to show how to solve this in Excel as well?

**You said:**

can you give 2 more examples for each above two sample

**ChatGPT said:**

Absolutely! Below are **two detailed examples** each for:

1. **Independent Two-Sample t-Test**
2. **Paired (Related Samples) t-Test**

Each example includes the setup, steps, and conclusion.

**🔹 PART 1: Independent Two-Sample t-Test Examples**

🧪 Use when comparing **two different groups** (no pairing between values)

**✅ Example 1: Equal Variance (Pooled t-test)**

**🔍 Scenario:**

A factory manager wants to compare the average production output of **two machines**.

| **Machine** | **n** | **Mean (X̄)** | **Std. Dev (s)** |
| --- | --- | --- | --- |
| A | 10 | 45 units | 3.5 |
| B | 12 | 42 units | 3.2 |

**Step-by-Step:**

* H₀: μ₁ = μ₂
* H₁: μ₁ ≠ μ₂
* Assume **equal variance**, α = 0.05
* df = 10 + 12 − 2 = 20
* Critical t (two-tailed) = ±2.086

**Pooled variance**:

Sp2=(9)(3.5)2+(11)(3.2)220=110.25+112.6420≈11.14*Sp*2​=20(9)(3.5)2+(11)(3.2)2​=20110.25+112.64​≈11.14

**Standard error**:

SE=11.14(110+112)≈11.14(0.1833)≈2.04≈1.43*SE*=11.14(101​+121​)​≈11.14(0.1833)​≈2.04​≈1.43

**t-statistic**:

t=45−421.43≈2.10*t*=1.4345−42​≈2.10

Since **2.10 > 2.086**, **reject H₀**

✅ **Conclusion**: The two machines produce **significantly different** outputs.

**✅ Example 2: Unequal Variance (Welch’s t-test)**

**🔍 Scenario:**

You want to compare the **average delivery time** of two couriers.

| **Courier** | **n** | **Mean Time** | **Std. Dev** |
| --- | --- | --- | --- |
| X | 15 | 24 mins | 2.5 |
| Y | 12 | 26 mins | 1.8 |

**Step-by-Step:**

* H₀: μ₁ = μ₂
* H₁: μ₁ ≠ μ₂
* Unequal variances → use **Welch’s t-test**

**Standard error**:

SE=2.5215+1.8212=6.2515+3.2412≈0.4167+0.27=0.6867≈0.8288*SE*=152.52​+121.82​​=156.25​+123.24​​≈0.4167+0.27​=0.6867​≈0.8288

**t-statistic**:

t=24−260.8288=−20.8288≈−2.41*t*=0.828824−26​=0.8288−2​≈−2.41

Use calculator for **df ≈ 24.5**, critical t ≈ ±2.064  
→ Since −2.41 < −2.064 → **Reject H₀**

✅ **Conclusion**: Delivery times are **significantly different** between couriers.

**🔹 PART 2: Paired (Related Samples) t-Test Examples**

🧪 Use when the **same subject** is measured twice (before/after, matched pairs)

**✅ Example 1: Training Impact on Sales**

**🔍 Scenario:**

5 employees attend a sales training. We measure **monthly sales** before and after.

| **Employee** | **Before** | **After** | **Difference (D)** |
| --- | --- | --- | --- |
| A | 1000 | 1300 | 300 |
| B | 1200 | 1250 | 50 |
| C | 1100 | 1200 | 100 |
| D | 1050 | 1100 | 50 |
| E | 950 | 1000 | 50 |

**Differences (D)** = [300, 50, 100, 50, 50]

* Dˉ=5505=110*D*ˉ=5550​=110
* SD of D = ≈ 100
* n=5*n*=5, df = 4

t=110100/5=11044.72≈2.46*t*=100/5​110​=44.72110​≈2.46

Critical t (α = 0.05, df = 4) = ±2.776 → **Do not reject H₀**

❌ **Conclusion**: No significant improvement in sales (not enough evidence).

**✅ Example 2: Blood Pressure Before and After Medication**

| **Patient** | **Before** | **After** | **Difference (D)** |
| --- | --- | --- | --- |
| P1 | 150 | 140 | -10 |
| P2 | 160 | 145 | -15 |
| P3 | 155 | 148 | -7 |
| P4 | 152 | 142 | -10 |
| P5 | 149 | 140 | -9 |

**Differences (D)** = [−10, −15, −7, −10, −9]

* Dˉ=−10.2*D*ˉ=−10.2
* SD of D ≈ 2.86
* n=5*n*=5, df = 4

t=−10.22.86/5=−10.21.278≈−7.98*t*=2.86/5​−10.2​=1.278−10.2​≈−7.98

Critical t (α = 0.05, df = 4) = ±2.776  
→ |−7.98| > 2.776 → **Reject H₀**

✅ **Conclusion**: The medication significantly reduced blood pressure.

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